Improper Integrals

Example

1. Calculate $\int_{-\infty}^{\infty} e^{-|x|} dx$.

Solution: We split it up at 0 to get

$$\int_{-\infty}^{\infty} e^{-|x|} dx = \int_{-\infty}^{0} e^{-|x|} dx + \int_{0}^{\infty} e^{-|x|} dx$$

$$= \lim_{t \to -\infty} \int_{t}^{0} e^{x} dx + \lim_{t \to \infty} \int_{0}^{t} e^{-x} dx = \lim_{t \to -\infty} e^{x}|_{t}^{0} + \lim_{t \to \infty} (-e^{-x})|_{0}^{t}$$

$$= 1 - 0 + 0 - (-1) = 2.$$

Problems

2. True   **FALSE** When calculating $\int_{-\infty}^{\infty} f(x) dx$, the final result depends on the a we choose to split it up as $\int_{-\infty}^{a} f(x) dx + \int_{a}^{\infty} f(x) dx$.

Solution: The answer does not depend on a.

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3. **TRUE**   False Since the function $f(x) = x$ is odd, we know that $\int_{-n}^{n} f(x) dx = 0$ for all integers n.

Solution: Since it is odd, we can think of the area left of 0 and the area right of 0 as canceling.

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4. True   **FALSE** Since the function $f(x) = x$ is odd, we know that $\displaystyle\int_{-\infty}^{\infty} f(x)dx = 0$ for

all integers n.

Solution: When we split up the integral, we get $\int_0^\infty x dx$ and $\int_{-\infty}^0 x dx$ which give ∞ and $-\infty$ respectively. Thus, the integral diverges.

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5. Compute $\displaystyle\int_{-\infty}^{\infty} 2xe^{-x^2}dx$.

Solution: We split it up to get

$$\int_{-\infty}^{\infty} 2xe^{-x^2} = \lim_{t\to-\infty}\int_t^0 2xe^{-x^2}dx + \lim_{t\to\infty}\int_0^t 2xe^{-x^2}dx = \lim_{t\to-\infty}\int_{t^2}^0 e^{-u}du + \lim_{t\to\infty}\int_0^{t^2} e^{-u}du$$

$$= \lim_{t\to-\infty} -e^0 + e^{-t^2} + \lim_{t\to\infty} -e^{-t^2} + e^0 = -1 + 1 = 0.$$

Histograms and Probability

Problems

6. Draw a histogram with bins $[0.5, 2.5], [2.5, 4.5], [4.5, 6.5], [6.5, 8.5], [8.5, 10.5]$.

7. Calculate the probability that randomly choosing a value between 1 and 10 gives 7 or 8.

Solution: Probability is good cases/all cases so $\frac{2}{10}$.

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8. Calculate the probability of getting a 7 or 8 using class data.

9. Calculate the probability that randomly choosing a value between 1 and 10 gives an even number.

Solution: Probability is good cases/all cases so $\frac{5}{10}$.

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10. Calculate the probability of getting an even number using class data.